ECE 515 / ME 540 **Assignment # 1**

Issued: Aug 24 Due: Aug 31, 2023

Reading Assignment:

BMP, Ch. 1.

Problems:

1. (Exercise 1.7.1 in BMP) Consider a nonlinear scalar input-output system whose input $u(t)$ and output $y(t)$ are related through the differential equation

$$\ddot{y} = 2y - (y^2 + 1)(\dot{y} + 1) + u.$$

   (i) Obtain a nonlinear state-space representation.

   (ii) Linearize this system of equations around the equilibrium trajectory when $u(\cdot) \equiv 0$ and write it down in state-space form.

2. Convert each of the following high-order differential equations into the input/state/output form:

   (i) $\ddot{y} - 2\dot{y} = 4u$

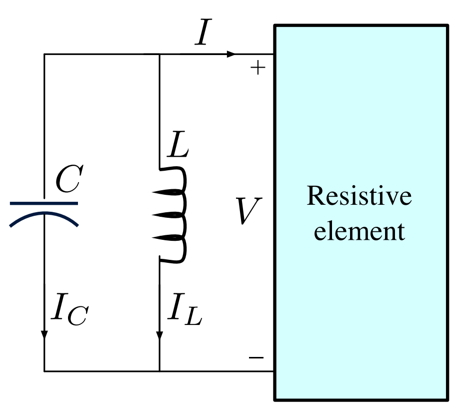
   (ii) $y^{(3)} + 2\dot{y} - 2y = -u$ ($y^{(3)}$ is the 3rd derivative of y with respect to time)

3. A nonlinear state-space model with no controls is a system of first-order ODEs that has the form

$$\dot{x}(t) = f(x(t)),$$

   where $x(t) \in \mathbb{R}^n$ is the state vector. The term "no controls" designates the fact that the external input u is absent, so the system evolves on its own. We say that a point $x_0 \in \mathbb{R}^n$ is an *equilibrium point* of the system if $f(x_0) = 0$.

   Consider the following circuit that contains linear components (an inductor and a capacitor) and a nonlinear resistive element:



   The voltage V across the resistive element and the current I flowing into it are related via a nonlinear voltage-current characteristic $I = g(V)$.

(i) Derive a second-order ODE for V. You may (and should) assume that g is differentiable.

(ii) Write down nonlinear state-space model with no controls for the ODE you have obtained in part (i).

(iii) Consider the following voltage-current characteristic:

$$g(V) = -V + \tfrac{1}{3}V^3.$$

Show that the zero state is the only eqiulibrium point of the state-space model from part (ii) and linearize the system around this equilibrium point.

4. (Exercise 1.7.9 in BMP) Consider the SISO LTI system with the transfer function

$$G(s) = \frac{s+4}{(s+1)(s+2)(s+3)}.$$

(i) Obtain a state-space representation in the controllable canonical form.

(ii) Now obtain one in the observable canonical form.

(iii) Use partial fraction expansion to obtain a representation with a diagonal state matrix A (modal canonical form).

5. (Exercise 1.7.10 in BMP) Repeat the above steps for the system with the transfer function

$$G(s) = \frac{s^3+2}{(s+1)(s+3)(s+4)}.$$