**ECE 498MR: Introduction to Stochastic Systems**

## Course Syllabus

**Catalog Description**: Exploration of noise, uncertainty, and randomness in the context of signals and systems. The course will introduce discrete- and continuous-time random processes as input and/or output signals of various types of systems, with and without memory or feedback. Probabilistic notions will be tightly integrated with techniques from signals and systems, such as linearity, time-invariance, causality, transform methods, and stability. Basic concepts will be illustrated via numerous examples, such as noise in linear and nonlinear circuits, average consensus and PageRank, queuing systems, noise in remote sensing applications, Bayesian filtering, Monte Carlo simulation, risk allocation in financial portfolios, stochastic gradient descent.

**Course Objectives:** Upon successful completion of the course, students will be able to reason about noise, uncertainty, and randomness in the context of engineering systems using tools and techniques from probability theory and systems theory.

**Target Audience:** Senior undergraduate and graduate students.

**Prerequisites:** ECE 210 and ECE 313.

**Coursework:** 50% homework (which will include regular programming assignments as iPython notebooks); 25% midterm; 25% final exam. Graduate students will earn an additional hour of credit by solving additional problems of a more theoretical nature.

**Text:** none required; instructor's lecture notes and iPython notebooks will be used. The following sources may be useful:

- E. A. Lee and P. Varaiya, *Structure and Interpretation of Signals and Systems*
- N. Wiener, *Nonlinear Problems in Random Theory*
- P. Albertos and I. Mareels, *Feedback and Control for Everyone*
- N. Gershenfeld, *The Nature of Mathematical Modeling* and *The Physics of Information Technology*

**Outline of Topics:**

1. Introduction: noise, uncertainty, and randomness in engineering systems (1 hr)

2. Review of signals and systems, probability (5 hrs)

3. First look: Markov chains as stochastic systems (4.5 hrs)
    a. Motivation: random walk on the integers -- matrix view
    b. Markov chains as nonlinear systems: $X[t+1] = f(X[t],Z[t])$
    c. Markov chains as linear systems in the space of probabilities: $p[t+1]=Kp[t]$
    d. Analysis by z-transform techniques; stability.
    e. Case studies: average consensus and PageRank.

4. Random signals and probabilistic systems (6 hrs)
    a. Random processes as signals (discrete- and continuous-time), in time and frequency domain.
    b. Events through Wiener's viewpoint (random waveform passing through "gates" to motivate independent increments, Markov property; inspiration for Feynman's path integral).

   c. Examples: random walk (revisited); Poisson process and other point processes; Wiener process; white and colored noise.

5. Following the dynamics (6 hrs)
   a. Moments, auto- and cross-correlation in time and frequency domain; spectral methods.
   b. Input-output relations; theorems of Bussgang and Campbell; fluctuation-dissipation relations.
   c. Basic analysis of convergence and stability via Lyapunov (or potential) functions.
   d. Case studies: average consensus and PageRank revisited.

6. Uncertainty (5 hrs)
   a. Dynamical view: evolution of uncertainty and information in time.
   b. Packet arrivals and departures in networks, queues as stochastic systems.
   c. A glimpse of Bayesian filtering.

7. Noise (3 hrs)
   a. Noise mechanisms in physical systems: shot noise, Johnson-Nyquist noise, van der Ziel (1/f) noise, amplifier noise.
   b. Case studies: discovery of cosmic microwave background radiation by Penzias and Wilson; noise and Bayesian filtering in remote sensing systems.

8. Randomness and determinism (5 hrs)
   a. Law of Large Numbers and the Central Limit Theorem through the lens of linear systems.
   b. Variance reduction by averaging (examples: invention of least squares; financial risk allocation in portfolios following Harry Markowitz; Monte Carlo simulation).
   c. Heuristic derivation of large-deviation bounds via Taylor series and Stirling approximation (example: probabilistic interpretation of multiplexing gain in telephony).

9. Feedback and control (5 hrs)
   a. Basics of controlled Markov chains: $X[t+1]=f(X[t],U[t],Z[t])$.
   b. Stabilization and optimization via feedback.
   c. Case study: stochastic gradient descent in machine learning.

10. Midterm (1.5 hrs)