**ECE 515 / ME 540** **Assignment # 7**

Issued: Nov 28 Due: Dec 5, 2023

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### Reading Assignment:

**BMP**, Ch. 10–11.

### Problems:

1. Consider the optimal control problem given by the system $\dot{x} = xu$ with $x \in \mathbb{R}$ and $u \in [-1, 1]$, no running cost ($\ell = 0$), and terminal cost $m(x) = x$. In other words, the cost functional is $J(u) = x(t_1)$. The final time $t_1$ is fixed and finite.

   (i) Find the optimal cost function (also called the value function) $V(t, x)$ by inspection (without using the HJB equation).

   (ii) Write down the HJB equation for this problem, and simplify it by computing the minimum in it.

   (iii) Does the optimal cost function from part (a) satisfy the HJB equation from part (b) everywhere?

2. (Exercise 11.6.1 in BMP) Consider the system $\dot{x} = u$ and the cost criterion

$$J(u) = \frac{q}{2}x^2(T) + \frac{1}{2}\int_0^T u^2(t)\,\mathrm{d}t$$

   with $x(0) = x_0$, the terminal time $T$ given, and $q \geq 0$. Find the optimal control using the Minimum Principle. First consider the case when $0 < q < \infty$. Then obtain the control for the limiting case "$q = \infty$," i.e., impose the constraint $x(T) = 0$. Do the controllers converge as $q \to \infty$?

3. (Exercise 11.6.2 in BMP) A simplified model for a reactor is given by the bilinear system $\dot{x} = xu$, where $x$ is the concentration and $u$ is the reaction rate taken as the control variable. Suppose that $x(0) = 0.5$ and the terminal time is $t_1 = 1$. Using the Minimum Principle, find the control function that minimizes the cost

$$J(u) = [x(1) - 1]^2 + \int_0^1 u^2(t)\,\mathrm{d}t.$$