Reading Assignment:
BMP, Ch. 1.

Problems:

1. (Exercise 1.7.1 in BMP) Consider a nonlinear scalar input-output system whose input \( u(t) \) and output \( y(t) \) are related through the differential equation

\[
\ddot{y} = 2y - (y^2 + 1)(\dot{y} + 1) + u.
\]

(i) Obtain a nonlinear state-space representation.
(ii) Linearize this system of equations around the equilibrium trajectory when \( u(\cdot) \equiv 0 \) and write it down in state-space form.

2. Convert each of the following high-order differential equations into the input/state/output form:

   (i) \( \ddot{y} - 2\dot{y} = 4u \)
   (ii) \( y^{(3)} + 2\dot{y} - 2y = -u \) (\( y^{(3)} \) is the 3rd derivative of \( y \) with respect to time)

3. A nonlinear state-space model with no controls is a system of first-order ODEs that has the form

\[
\dot{x}(t) = f(x(t)),
\]

where \( x(t) \in \mathbb{R}^n \) is the state vector. The term “no controls” designates the fact that the external input \( u \) is absent, so the system evolves on its own. We say that a point \( x_0 \in \mathbb{R}^n \) is an equilibrium point of the system if \( f(x_0) = 0 \).

Consider the following circuit that contains linear components (an inductor and a capacitor) and a nonlinear resistive element:

The voltage \( V \) across the resistive element and the current \( I \) flowing into it are related via a nonlinear voltage-current characteristic \( I = g(V) \).
(i) Derive a second-order ODE for $V$. You may (and should) assume that $g$ is differentiable.

(ii) Write down nonlinear state-space model with no controls for the ODE you have obtained in part (i).

(iii) Consider the following voltage-current characteristic:

$$g(V) = -V + \frac{1}{3}V^3.$$ 

Show that the zero state is the only equilibrium point of the state-space model from part (ii) and linearize the system around this equilibrium point.

4. (Exercise 1.7.9 in BMP) Consider the SISO LTI system with the transfer function

$$G(s) = \frac{s + 4}{(s + 1)(s + 2)(s + 3)}.$$ 

(i) Obtain a state-space representation in the controllable canonical form.

(ii) Now obtain one in the observable canonical form.

(iii) Use partial fraction expansion to obtain a representation with a diagonal state matrix $A$ (modal canonical form).

5. (Exercise 1.7.10 in BMP) Repeat the above steps for the system with the transfer function

$$G(s) = \frac{s^3 + 2}{(s + 1)(s + 3)(s + 4)}.$$