ECE 534 RANDOM PROCESSES **FALL 2012**
PROBLEM SET 4 **Due beginning of class, Tuesday, November 13**

4. Random Processes

Assigned Reading: Chapter 4 of the course notes.

Problems to be handed in:

Problems 4.3, 4.11, 4.13, 4.25, 4.27 from the course notes, as well as the following three problems:

1 Two random processes

Consider the probability space $(\Omega, \mathcal{F}, P)$ with $\Omega = \mathbb{R}$, $\mathcal{F} = \mathcal{B}(\mathbb{R})$ (the Borel σ-algebra on the reals) and P having the pdf of a Unif$[0,1]$ random variable.

(a) Define the random process $X = (X_t)_{t \in [0,\infty)}$ by

$$X_t(\omega) = \begin{cases} 1, & \text{if } 0 < t \le \omega \\ 0, & \text{otherwise} \end{cases}$$

Find the pmf p_{X_t} for each $t \in [0, \infty)$.

(b) Define another random process $Y = (Y_t)_{t \in [0,\infty)}$ by

$$Y_t(\omega) = \begin{cases} t/\omega, & \text{if } 0 < t \le \omega \\ 0, & \text{otherwise} \end{cases}$$

For each $t \in [0, \infty)$ and each $y \in (0, 1)$, find $P(Y_t > y)$.

2 Standardized random walk

Let $X = (X_k)_{k \in \mathbb{Z}_+}$ be an i.i.d. process with each X_k taking values ± 1 with equal probability. For each $n \in \mathbb{Z}_+$, define the standardized sum

$$Y_n = \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} X_k.$$

Show that Y_n has the characteristic function

$$\Phi_{Y_n}(u) = e^{n \log \cos(u/\sqrt{n})}.$$

Find the limit $\lim_{n\to\infty} \Phi_{Y_n}(u)$ for each $u \in \mathbb{R}$.

3 Multiplication by random signs

Let $X = (X_n)_{n \in \mathbb{Z}_+}$ be an i.i.d. Gaussian process with mean zero and autocorrelation function $R_X(0) = \sigma^2$. Let $U = (U_n)_{n \in \mathbb{Z}_+}$ be an i.i.d. random process with $P(U_n = -1) = P(U_n = 1) = 1/2$ for all n. Assume that the processes X and U are independent of each other. Define a new random process $Y = (Y_n)_{n \in \mathbb{Z}_+}$ by $Y_n = U_n X_n$.

(a) Find the autocorrelation function $R_Y(k, l)$.

(b) Find the characteristic function Φ_{Y_n} for each n.

(c) Is Y an i.i.d. process? Why or why not?

(d) Do the sample averages

$$\overline{Y}_n = \frac{1}{n}\sum_{k=0}^{n-1} Y_k$$

converge in the mean square sense? If so, to what?