Lecture I: Introduction

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This lecture

In this first lecture, we will take a bird's eye view of signals and systems.

Don't worry if things seem a bit abstract or weird at this point: we will fill in all the details as the semester goes on. This just gives you an idea of what to expect and what kinds of things we will be looking at.

Some prerequisites for the class (we will review them when necessary):

- complex numbers: arithmetic operations, geometric representation, Euler's formula
- working knowledge of linear circuits (Ohm's law, Kirchoff laws, RLC stuff, etc.)
- basic knowledge of linear differential equations
- basic knowledge of linear algebra: matrices, eigenvectors/eigenvalues, linear independence, etc.

What is a signal?

A **signal** is any real-valued function that changes in time and/or space.

Signals that vary in time:

- neural spike train recordings
- speech or audio waveforms (and their sampled versions)
- EEG or ECG readings
- stock prices
- social trend data (e.g., `http://trends.google.com`)

Signals that vary in space:

- digital images
- CT or MRI scans

Signals that vary both in time and in space:

- video

We will mostly be concerned with time-varying signals in this class.

Continuous-time and discrete-time signals

Continuous-time (**analog**) signals: functions $x(t)$, where $-\infty < t < \infty$

- dc signal: $x(t) = A$ for all t (the most boring signal)
- sinusoids: $x(t) = A\sin(\omega t + \theta)$
- other complicated functions: $x(t) = Ae^{-Kt}\sin(\omega t + \theta) - \cosh(Lx)$
- ...

Discrete-time signals: sequences $x[n]$, where n is an integer

- constant signal: $x[n] = A$ for all n (also very boring)
- binary signals: $x[n] \in \{0, 1\}$ for all n
- other complicated signals: e.g., **sampled** analog signals $x[n] = f(nT)$, where $f(t)$ is some continuous-time signal, and T is the *sampling period*

What is a system?

A **system** is any device, process or computer program that converts signals into other signals.

Mathematically:

$$y = S\{x\}$$

The notation $S\{\cdot\}$ is meant to convey the fact that a system is a transformation that maps **functions** into other **functions**. More precisely,

$$y(t) = S\Big\{x(t)\Big\} \qquad \text{for continuous-time signals}$$

$$y[n] = S\Big\{x[n]\Big\} \qquad \text{for discrete-time}$$

Our goals

In this class, we will learn how to:

- **represent** various types of signals
- **model** various types of systems
- **analyze** various systems and understand how they transform signals of interest to us
- **synthesize** systems with desired properties

Obviously, system analysis and synthesis can be extremely difficult, unless some simplifying assumptions are made. In this class, we will see that a wide variety of useful system models satisfy two such key assumptions: **linearity** and **time-invariance**.

Linearity

Signals can be

- **added**: if $x_1(t)$ and $x_2(t)$ are signals, then so is $x(t) = x_1(t) + x_2(t)$
- **scaled**: if $\alpha \in \mathbb{R}$ and $x_0(t)$ is a signal, then so is $x(t) = \alpha x_0(t)$

The same goes for discrete-time signals.

A **linear system** is a system that respects these operations:

- if $y_1(t) = S\Big\{x_1(t)\Big\}$ and $y_2(t) = S\Big\{x_2(t)\Big\}$, then

$$y(t) = S\Big\{x(t)\Big\} = S\Big\{x_1(t)\Big\} + S\Big\{x_2(t)\Big\} = y_1(t) + y_2(t)$$

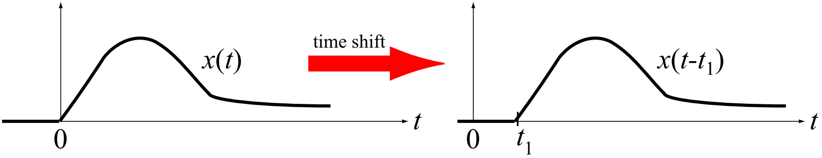
- if $y_0(t) = S\Big\{x_0(t)\Big\}$, then

$$y(t) = S\Big\{x(t)\Big\} = \alpha S\Big\{x_0(t)\Big\} = \alpha y_0(t)$$

In this class, we will deal primarily with linear systems.

Time invariance

Signals can be **shifted in time**:



A **time-invariant system** is a system that respects the time-shift operation: if

$$y(t) = S\Big\{x(t)\Big\},$$

then

$$S\Big\{x(t-t_1)\Big\} = y(t-t_1).$$

That is, a time-shifted version of the input leads to the time-shifted version of the output.

Linear time-invariant (LTI) systems

Why bother with linearity and time-invariance?

Two reasons:

- many physical, engineering and biological systems can be modeled (at least to a good approximation) by LTI systems
- LTI systems are particularly easy to analyze

LTI systems can be implemented using electric circuits containing resistors, inductors and capacitors (RLC circuits)

Linear system models: I

Linear differential and difference equations — often written down from "physical" considerations

- continuous-time:

$$C\frac{dy(t)}{dt} + \frac{1}{R}y(t) = x(t)$$

  (parallel RC circuit)

- discrete-time:

$$y[n] = \frac{x[n-1] + x[n] + x[n+1]}{3}$$

  (3rd order moving-average filter)

These are simple cases; in general, LDEs can have time-varying coefficients and must be solved numerically.

Linear system models: II

Convolution representation

- continuous-time:

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)x(\tau)d\tau$$

- discrete-time:

$$y[n] = \sum_{k=-\infty}^{\infty} h[n,k]x[k]$$

Don't worry for now what the h is, we will get into these details later. For now, note the similarity to matrix multiplication:

$$y = Ax, \qquad x \in \mathbb{R}^s, A \in \mathbb{R}^{r \times s}$$

$$y[n] = \sum_{k=1}^{s} A[n,k]x[k]$$

This is **not accidental!**

Linear system models: III

Frequency-domain representation

We can represent sufficiently "regular" signals by their **frequency content**:

$$x(t) \quad \longleftrightarrow \quad X(\omega)$$

The idea is to use sums (or integrals) of complex exponentials, such as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \qquad j = \sqrt{-1}$$

As we will see, LTI systems act on complex exponentials in a very special way: if S is LTI, then

$$S\left\{e^{j\omega t}\right\} = c(\omega)e^{j\omega t},$$

where $c(\omega)$ is some complex number that depends only on ω. By linearity,

$$S\left\{x(t)\right\} = S\left\{\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right\} = \left\{\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right\} = \sum_{k=-\infty}^{\infty} a_k c(\omega) e^{jk\omega_0 t}.$$

We will look at these methods in great detail.

System analysis in frequency domain is (generally) much easier than system analysis in time domain.

A bunch of important system components are naturally viewed in the frequency domain:

- filters — remove or attenuate unwanted frequencies; amplify or ehnance desired frequencies

Nonlinear systems are systems too!

Although our emphasis will be on linear systems, we shouldn't neglect nonlinear ones.

Examples of nonlinear systems:

- active electronic circuits containing diodes, transistors, etc.
- biological and artificial neural nets
- the stock market

Sometimes, nonlinear systems can be well approximated by their "linearized" versions — e.g., for weak input signals.

A fun example: van Eck phreaking

In 1985, Wim van Eck published a paper arguing that it is possible to eavesdrop on the contents of a CRT display by detecting its electromagnetic radiation (see Wikipedia entry on "van Eck phreaking")

We can represent this as a cascade of **nonlinear** systems

$$x[n] \xrightarrow{S_1} y[k,l,n] \xrightarrow{S_2} z(\mathbf{w},t) \xrightarrow{S_3} \widehat{y}[k,l,n] \xrightarrow{S_4} \widehat{x}[n],$$

where:

- x represents a typed text message; $x[n]$ is the nth symbol and can take 27 possible values (assuming only capital letters and spaces)
- y represents the state of the CRT display; $y[k,l,n]$ is the intensity of the pixel at location (k,l) when the nth symbol is typed
- z represents the EM radiation emitted by the display; $z(\mathbf{w},t)$ is the EM field at point $\mathbf{w}$ in space at time t (this is a continuous-time and space signal!)
- $\widehat{y}$ represents the decoded contents of the display
- $\widehat{x}$ represents the decoded contents of the text message