BME 171, Sec 2: Homework 3 **due Tue, Sep 23 by 5pm**

Note: in some of these problems, you are asked to use MATLAB to plot certain signals. Turn in all your MATLAB code together with printouts of your plots with axes clearly labeled, just as in Kamen and Heck. Make sure that it is clear from the printout which plot corresponds to which problem.

1. **Linear filtering of noisy signals.** In this problem, you will investigate the use of linear filters for removing noise from discrete-time signals. Consider a discrete-time signal of the form

$$x[n] = s[n] + z[n], \qquad n = \ldots, -2, -1, 0, 1, 2, \ldots$$

where $s[n]$ is some *unknown*, sufficiently "smooth" function of n and $z[n]$ is a random noise signal. Without getting too deep into random signals (which are not the topic of this course), we assume that the noise $s[n]$ is, on average, equal to zero and that it is *independent* of the "clean" signal $s[n]$. At each time step $n = 0, 1, 2, \ldots$, we have access to N most recent observations $x[n], \ldots, x[n-N+1]$, and we use these observations as input to a moving-average (MA) filter

$$y[n] = \sum_{i=0}^{N-1} a_i x[n-i],$$

where $a_0, \ldots, a_{N-1}$ are the filter coefficients. In this problem, we will look at *exponentially weighted moving-average (EWMA)* filters. Such a filter has the form

$$y[n] = \frac{1-b}{1-b^N} \sum_{i=0}^{N-1} b^i x[n-i],$$

for some $0 < b < 1$. Thus, we have

$$a_i = \frac{1-b}{1-b^N} b^i, \qquad i = 0, \ldots, N-1.$$

Let us now consider the case when

$$s[n] = 5 \sin(2\pi n/20 + 3) u[n],$$

where $u[n]$ denotes the discrete-time unit step signal. The noise $z[n]$ is assumed to be a Gaussian white noise sequence with unit variance. To generate such a sequence of length m, you can use the MATLAB command `randn(1,m)`; this will create a row vector of length m.

(a) Use MATLAB to plot the corresponding noisy signal $x[n] = s[n] + z[n]$ for $n = 0, 1, \ldots, 60$.

(b) Now let $b = 0.2$ and $N = 3$. Assuming that at $n = 0$ we initialize the EWMA filter with $x[-1] = \ldots = x[-N] = 0$, plot the output of the filter for $n = 0, 1, \ldots, 60$.

(c) Repeat part (b) with $b = 0.2$ and $N = 10$.

(d) Comment on the relationship between the filter outputs in parts (b) and (c) with the original "clean" signal $s(n)$.

(e) What do you think is the effect of increasing b? What about increasing N?

2. **Textbook problems.** Work problems 2.5, 2.6, 2.7, 2.9(a)-(c), 2.29, 2.31, 2.33 from Kamen and Heck.