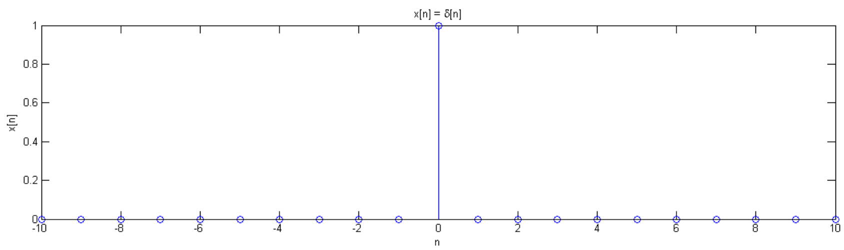
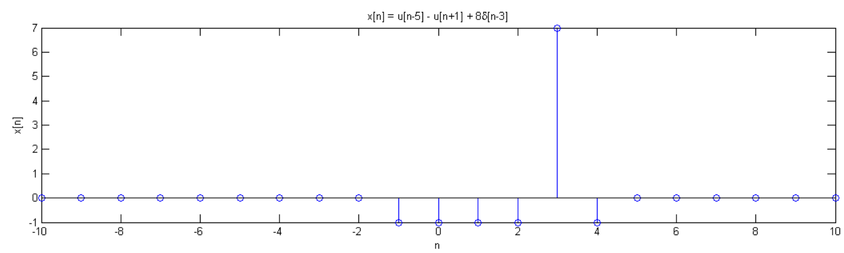
BME 171 Fall 2008
Homework 1 Solutions
100 points total

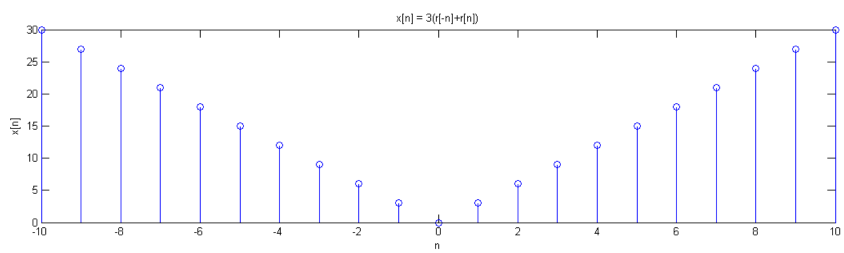
Problem 1 – Part A

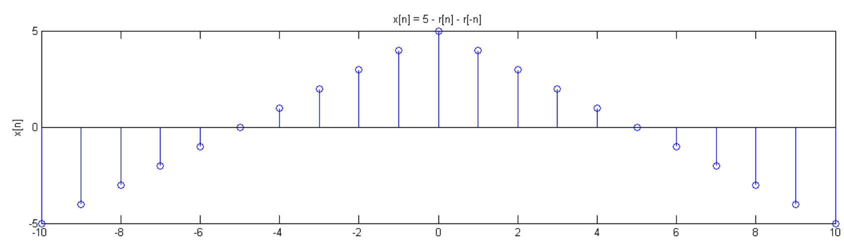
20 points → 4 points per plot for correct line shape, 1 point per plot for labeled axes
→Any code that generates the requested plots is acceptable

Correct figures:









Example of code:

```
n = -10:10;
for k = 1:length(n)
    %% code for part A
    a(k) = 0;
    if n(k) == 0
        a(k) = 1;
```

```
    end

    %% code for part B
    b(k) = 0;
    if n(k)-5 >= 0
        b(k) = b(k) + 1;
    end
    if n(k)+1 >=0
        b(k) = b(k) - 1;
    end
    if n(k)-3 == 0
        b(k) = b(k) + 8;
    end

    %% code for part C
    c(k) = 0;
    if -n(k) >= 0
        c(k) = c(k) - 3*n(k);
    end
    if n(k) >= 0
        c(k) = c(k) + 3*n(k);
    end

    %% code for part D
    d(k) = 5;
    if n(k) >= 0
        d(k) = d(k) - n(k);
    end
    if -n(k) >= 0
        d(k) = d(k) + n(k);
    end
end

figure(1)
subplot(2,1,1)
stem(n,a)
title('x[n] = \delta[n]')
xlabel('n')
ylabel('x[n]')

subplot(2,1,2)
stem(n,b)
title('x[n] = u[n-5] - u[n+1] + 8\delta[n-3]')
xlabel('n')
ylabel('x[n]')

figure(2)
subplot(2,1,1)
stem(n,c)
title('x[n] = 3(r[-n]+r[n])')
xlabel('n')
ylabel('x[n]')

subplot(2,1,2)
stem(n,d)
title('x[n] = 5 - r[n] - r[-n]')
xlabel('n')
ylabel('x[n]')
```

Problem 1 – Part B

10 points, 3 points for each analytical solution and 2 points for each plot

No credit awarded for analytical solutions with incorrect conclusions (i.e. periodic vs non-periodic)

A signal x(t) is periodic if for some period T the following holds:

$$x[n+T] = x[n]$$

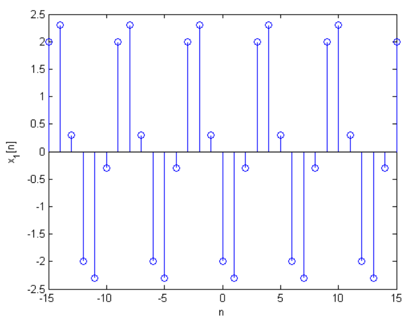
Applying the form $x[n] = Asin(\omega n + \phi)$ and recalling the formula $T = {}^{2\pi}/_{\omega}$, for x_1 we have:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/3} = 6$$

$$x[n+T] = 2.5\cos(\frac{(n+6)\pi}{3} + 2.5) = 2.5\cos(2\pi + \frac{n\pi}{3} + 2.5) = x[n]$$

<u>Thus the signal is periodic with a period of 6 samples.</u>

This is also clearly shown from a plot of x_1:



Code:

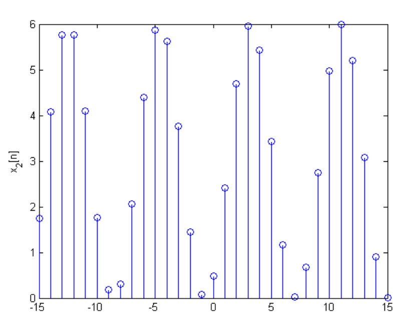
```
n = -15:15;
x1 = 2.5*cos(n*pi./3 + 2.5);
stem(n,x1)
xlabel('n')
ylabel('x_1[n]')
```

Similar analysis can be applied to x_2 as follows:

$$T = \frac{2\pi}{f} = \frac{2\pi}{4/5} = \frac{5\pi}{2}$$

Since the calculated period is not an integer number of samples, **<u>the signal is NOT periodic.</u>**

This is also clearly shown in the following plot of x_2:



Code:

```
n = -15:15;
x2 = 3*sin(4*n/5-1)+3;
stem(n,x2)
xlabel('n')
ylabel('x_2[n]')
```

Problem 2 – Part A
10 points as indicated

Prove that $e^{j\theta} = \cos\theta + j\sin\theta$

The Taylor expansions for each term are as follows:

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, x^{2n+1}$
$\cos \mathrm{x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{2n}$ (1 point)

Substituting in jθ for x:
$e^{j\theta} = 1 + j\theta - \theta^2/_{2!} - j\theta^3/_{3!} + \theta^4/_{4!} + j\theta^5/_{5!} \cdots$ (2 points)

$\mathrm{j}\sin\theta = j\left(\theta - \theta^3/_{3!} + \theta^5/_{5!}\right) \cdots$ (2 points)
$\cos\theta = 1 - \theta^2/_{2!} + \theta^4/_{4!} \cdots$ (2 points)

Rearranging the expression for $e^{j\theta}$ yields:
$e^{j\theta} = \left(1 - \theta^2/_{2!} + \theta^4/_{4!} - \cdots\right) + j\left(\theta - \theta^3/_{3!} + \theta^5/_{5!} - \cdots\right)$ (2 points)
$e^{j\theta} = \cos\theta + j\sin\theta$ QED (1 point)

Problem 2, Part B

10 points as indicated

From the hint, compute the complex conjugate of Euler's identity:

$e^{-j\theta} = \cos -\theta + j\sin -\theta = \cos\theta - j\sin\theta$ (4 points)

Now notice that:

$e^{j\theta} + e^{-j\theta} = 2\cos\theta$ (2 points)

$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$ (2 points)

And thus:

$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$ (1 point)

$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$ (1 point)

Alternative solution using series expansions:

From the hint, evaluate the complex conjugate of $e^{j\theta}$, i.e.:

$e^{-j\theta} = 1 - j\theta - \theta^2/2! - j\theta^3/3! - \theta^4/4! - j\theta^5/5! \cdots$ (4 points)

Observe that for the CC, all terms in the series become negative.

Also, remember from above that:

$e^{j\theta} = 1 + j\theta - \theta^2/2! - j\theta^3/3! + \theta^4/4! + j\theta^5/5! \cdots$

Recall that:

$\sin\theta = \theta - \theta^3/3! + \theta^5/5! \cdots$

$\cos\theta = 1 - \theta^2/2! + \theta^4/4! \cdots$

Further notice that:

$e^{j\theta} + e^{-j\theta} = 2\left(1 - \theta^2/2! + \theta^4/4! \cdots\right) = 2\cos\theta$ (2 points)

$e^{j\theta} - e^{-j\theta} = 2j\left(\theta - \theta^3/3! + \theta^5/5! \cdots\right) = 2j\sin\theta$ (2 points)

And thus:

$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$ (1 point)

$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$ (1 point)

Problem 2, Part C
10 points, as indicated

Prove the following:

$$\frac{1}{2}\int_{-1}^{1} e^{j\pi mt}dt = \begin{cases} 1 \text{ for m} = 0 \\ 0 \text{ for m} = \pm1, \pm2, \pm3, \ldots \end{cases}$$

Integrate:

$$= \frac{1}{2}\left[\frac{e^{j\pi mt}}{j\pi m}\right]_{-1}^{1}$$ (2 points)

Expand:

$$= \frac{e^{j\pi m} - e^{-j\pi m}}{2j\pi m}$$ (2 points)

Applying the identity for sinθ from part B, we have:

$$= \frac{\sin(\pi m)}{\pi m}$$ (2 points)

Finally, apply the definition of a sinc function

$$= \text{sinc(m)} = \begin{cases} 1 \text{ for m} = 0 \\ 0 \text{ for m} = \pm1, \pm2, \pm3, \ldots \end{cases}$$ QED (2 points)

Kamen & Heck 1.1

20 points – 2 points for each equation and 2 points for each plot

Let $\text{tri}_\tau(t) = (1 - \frac{2|t|}{\tau})p_\tau(t)$. The formula for each plot can then be found by inspection:

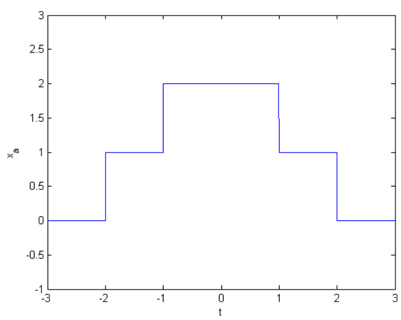
a) $x(t) = p_2(t) + p_4(t)$

b) $x(t) = \frac{4}{3}\text{tri}_8(t) - \frac{1}{3}\text{tri}_2(t)$

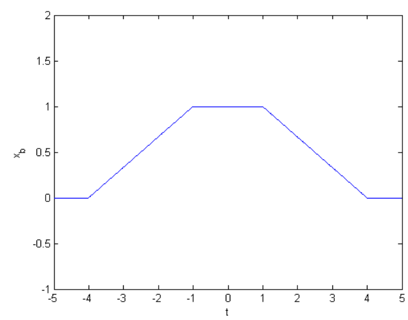
c) $x(t) = 2[p_{12}(t) + p_6(t) + \text{tri}_6(t)]$

d) $x(t) = 4\,p_4(t) - 2\,\text{tri}_4(t)$

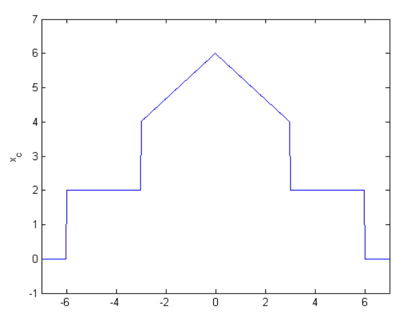
e) $\sum_{n=0}^{\infty} p_1(t + 2n + 0.5)$



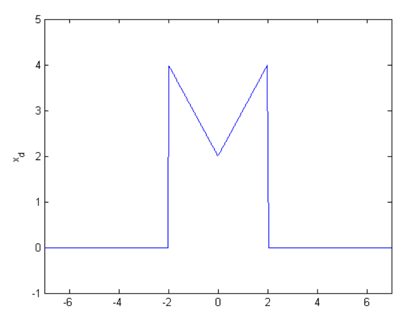
a)



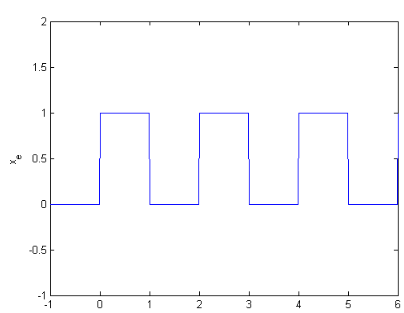
b)



c)



d)



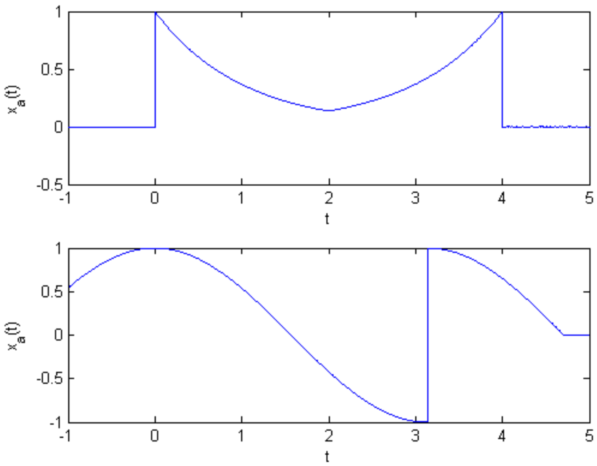
e)

Example code:

```
t = linspace(-3,3,1000);
a = zeros(1,length(t));
for k = 1:length(t)
    if t(k) >= -2 & t(k) < 2
        a(k) = a(k) + 1;
    end
    if t(k) >= -1 & t(k) < 1
        a(k) = a(k) + 1;
    end
end
figure(1); plot(t,a); xlabel('t'); ylabel('x_a'); axis([-3 3 -1 3])

t = linspace(-5,5,1000); b = zeros(1,length(t));
for k = 1:length(t)
    if t(k) >= -4 & t(k) < 4
        b(k) = b(k) + 4/3*(1-2*abs(t(k))/8);
    end
    if t(k) >= -1 & t(k) < 1
        b(k) = b(k) - 1/3*(1-2*abs(t(k))/2);
    end
end
figure(2); plot(t,b); xlabel('t'); ylabel('x_b'); axis([-5 5 -1 2])

t = linspace(-7,7,1000); c = zeros(1,length(t));
for k = 1:length(t)
    if t(k) >= -6 & t(k) < 6
        c(k) = c(k) + 2;
    end
    if t(k) >= -3 & t(k) < 3
        c(k) = c(k) + 2;
    end
    if t(k) >= -3 & t(k) < 3
        c(k) = c(k) + 2*(1-2*abs(t(k))/6);
    end
end
figure(3); plot(t,c); xlabel('t'); ylabel('x_c'); axis([-7 7 -1 7])

t = linspace(-7,7,1000); d = zeros(1,length(t));
for k = 1:length(t)
    if t(k) >= -2 & t(k) < 2
        d(k) = d(k) + 4;
    end
    if t(k) >= -2 & t(k) < 2
        d(k) = d(k) - 2*(1-2*abs(t(k))/4);
    end

end
figure(4); plot(t,d); xlabel('t'); ylabel('x_d'); axis([-7 7 -1 5])

t = linspace(-1,6,1000); e = zeros(1,length(t));
for n = 0:3
    for k = 1:length(t)
        if t(k) >= 2*n & t(k) < 2*n+1
            e(k) = e(k) + 1;
        end
    end
end
figure(5); plot(t,e); xlabel('t'); ylabel('x_e'); axis([-1 6 -1 2])
```

Kamen and Heck 1.5

10 points, 4 points per plot, 1 point for labeled axes



Example code:

```
t = linspace(-1,5,1000);
ut = zeros(1,length(t));
utminus2 = zeros(1,length(t));
utmins4 = zeros(1,length(t));
utpluspiover2 = zeros(1,length(t));
utminuspi = zeros(1,length(t));
utminus3piover2 = zeros(1,length(t));
for k = 1:length(t)
    if t(k) >=0
        ut(k) = 1;
    end
    if t(k) >= 2
        utminus2(k) = 1;
    end
    if t(k) >= 4
        utminus4(k) = 1;
    end
    if t(k) >= -pi/2
        utpluspiover2(k) = 1;
    end
    if t(k) >= pi
        utminuspi(k) = 1;
    end
    if t(k) >= 3*pi/2
        utminus3piover2(k) = 1;
    end
end
x_a = exp(-t).*ut + exp(-t).*(exp(2*t-4)-1).*utminus2 - exp(t-4).*utminus4;
subplot(2,1,1)
plot(t,x_a)
xlabel('t')
ylabel('x_a(t)')
x_b = cos(t) .* (utpluspiover2 - 2*utminuspi) + cos(t) .* utminus3piover2;
subplot(2,1,2)
plot(t,x_b)
xlabel('t')
ylabel('x_b(t)')
```

Kamen and Heck 1.6
10 points as indicated

$x(t)u(t-c) = v(t-c)u(t-c)$

Note that you cannot divide both sides by $u(t-c)$ as you would be dividing by zero for all times $t < c$. Therefore you must evaluate each condition separately:

For $t < c$, both sides of the equation are zero and the problem is trivial (2 points)

For $t \geq c$, $u(t-c) = 1$ and thus we have:

$x(t) = v(t-c)$ (1 point)

Thus, $v(t)$ is simply $x(t)$ delayed by a time c. Consequently, $x(t)$ is just $v(t)$ advanced by c, i.e.:

$v(t) = x(t+c)$ (1 point)

Part i

$x(t) = e^{-2t}$ where $c = 3$
$v(t) = x(t+3) = e^{-2(t+3)}$
$v(t) = e^{-6}e^{-2t}$ (2 points)

Part ii

$x(t) = t^2 - t + 1$ where $c = 2$
$v(t) = x(t+2) = (t+2)^2 - (t+2) + 1$
$v(t) = t^2 + 3t + 3$ (2 points)

Part iii

$x(t) = \sin(2t)$ where $c = \pi/4$
$v(t) = x(t + \pi/4) = \sin\left(2\left(t + \frac{\pi}{4}\right)\right) = \sin(2t + \frac{\pi}{2})$
$v(t) = \cos(2t)$ (2 points)

Note, all of these solutions are only valid for $t > c$