LETTER TO THE EDITOR

Generation and manipulation of squeezed states of light in optical networks for quantum communication and computation

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Abstract
We analyse a fiber-optic component which could have multiple uses in novel
information processing systems utilizing squeezed states of light. Our
approach is based on the phenomenon of photon-number squeezing of
soliton noise after the soliton has propagated through a nonlinear optical
fiber. Application of this component in optical networks for quantum
computation and quantum cryptography are discussed.

Keywords: Soliton squeezing, squeezing in fibers, quantum states, quantum
communication, quantum cryptography

Information processing systems where information is carried
by nonclassical states of light [1] (e.g., photon-number states
or squeezed states), though not as yet implemented with
standard telecommunications in mind, nevertheless present an
attractive alternative for such novel applications as quantum
computation [2] or secure quantum-key distribution [3]. As
currently envisioned [2, 4], such systems can be constructed
using linear optics, linear mixing of creation and annihilation
operators (linear Bogoliubov transformations which include
squeezing as a special case), and nonlinear operations for state
preparation and detection.

One major obstacle to current practical implementation of
such systems is the lack of components which would preserve
the noise characteristics of the transmitted information. Such
components would be crucial because squeezed states are
highly sensitive to loss, as a simple analysis reveals. In optical
communication networks, optical amplifiers are inserted along
the fiber links to restore exponentially decaying signal power
to acceptable levels. These devices, represented by an
idealized model of a phase-insensitive amplifier (PIA), add
at least 3 dB of noise with coherent-state inputs when the
amplifier gain is high [5] (this minimal noise figure is referred

to as the standard quantum limit, or SQL[1]) and thus may
destroy precisely engineered noise statistics of the transmitted
states. The quantum-key distribution scheme of Gottesman
and Preskill [3], for example, calls for at least 2.51 dB of
squeezing in channels with weak noise, so introducing a
PIA into the underlying communications infrastructure may
severely affect the performance of the scheme.

In this letter we concentrate on a component of such
information processing systems that could prove useful for
generating and manipulating squeezed states of light, in
particular photon-number squeezed (PNS) states. As recent
experimental reports indicate [6], it is possible to generate sub-
Poissonian light (a near-PNS state) by means of the following
simple set-up. A soliton pulse is launched into an optical fiber
and then frequency-filtered at the fiber exit. The resulting
quantum-mechanical state of light exhibits photon-number
fluctuations below the coherent-state level, with maximum

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[1] The term 'standard quantum limit' (SQL) arises in the literature on quantum
measurements (e.g., position monitoring of a free mass) and is usually taken to
signify some sort of a fundamental bound imposed by the quantum-mechanical
nature of the problem at hand. In the context of quantum amplifiers, the SQL
refers to the ultimate lower bound on the noise figure of a high-gain PIA with
coherent-state inputs.

squeezing observed when the fiber is three soliton periods long. To illustrate the utility of this set-up to our goal, we analyse the arrangement in which the above process is iterated by launching the filtered soliton through another fiber-and-filter stage. From here on, we will refer to our set-up as the dual-stage squeezer (DSS), as opposed to the single-stage squeezer (SSS) described at the beginning of the paragraph.

Our motivation to suggest the use of the DSS in optical networks for quantum information processing comes from the observation, further elaborated below, that the DSS can be thought of as the original SSS operating on squeezed (rather than simply coherent) input states. This property of the DSS makes it a natural choice for easily implementable sources of squeezed states, whose degree of squeezing can be controlled by tuning the soliton-pulse parameters, and also for devices that enhance the degree of squeezing introduced into the quantum network. For reasons mentioned above, such squeezing enhancers would conceivably play a key role in practical implementations of the Gottesman–Preskill quantum key distribution scheme using squeezed states [3].

In a recent study, Levandovsky *et al* [7] employed the soliton perturbation approach [8] to obtain a complete theoretical description of the quantum-noise statistics of spectrally filtered solitons. This linearization approach is valid whenever the photon-number noise is small compared to the average number of photons in the soliton, which is usually the case in most experiments. We briefly recount the main idea of their approach and then apply it to our analysis of the DSS.

Nonlinear evolution of an electromagnetic pulse propagating through a lossless optical fiber is governed by the quantum nonlinear Schrödinger equation (NLSE)

$$\frac{\partial}{\partial \xi} \hat{a}(\tau, \xi) = i \left[\frac{1}{2} \frac{\partial^2}{\partial \tau^2} + \hat{a}^\dagger(\tau, \xi) \hat{a}(\tau, \xi) \right] \hat{a}(\tau, \xi), \quad (1)$$

where $\hat{a}(\tau, \xi)$ is the annihilation operator of the field and (ξ, τ) are the dimensionless space and time coordinates. The corresponding classical equation has a fundamental soliton solution $a_0(\tau, \xi) = e^{i\xi/2} \mathrm{sech}\,\tau \equiv f_0(\tau) e^{i\xi/2}$, given here in canonical form with two photons per pulse. We write the annihilation operator as

$$\hat{a}(\tau, \xi) = [f_0(\tau) + \Delta \hat{a}(\tau, \xi)] e^{i\xi/2}, \quad (2)$$

where $\Delta \hat{a}$ is the annihilation operator that represents the perturbation of the soliton mean field by quantum noise and satisfies the usual equal-space commutation relations, $[\Delta \hat{a}(\tau, \xi), \Delta \hat{a}(\tau', \xi)] = [\Delta \hat{a}^\dagger(\tau, \xi), \Delta \hat{a}^\dagger(\tau', \xi)] = 0$, $[\Delta \hat{a}(\tau, \xi), \Delta \hat{a}^\dagger(\tau', \xi)] = \delta(\tau - \tau')$, everywhere inside the fiber. We make the linearization approximation by substituting equation (2) into equation (1) and discarding all terms that are $O(\Delta \hat{a}^2)$, thus separating the problem into the classical NLSE for the mean field $a_0(\tau, \xi)$ and the linearized operator equation

$$\frac{\partial \Delta \hat{b}}{\partial \xi} = \frac{i}{2} \frac{\partial^2}{\partial \tau^2} \Delta \hat{b} + 2i |a_0(\tau, \xi)|^2 \Delta \hat{b} + i a_0^2(\tau, \xi) \Delta \hat{b}^\dagger \quad (3)$$

for $\Delta \hat{b} = \Delta \hat{a} e^{i\xi/2}$. In the following, we shall disregard phase factors of the form $e^{i\theta\xi}$, $-\infty < \theta < \infty$, because the filtered light is directly detected in a SSS.

The solution of equation (3) can be written as an eigenfunction expansion [8]

$$\Delta \hat{a}(\tau, \xi) = \frac{1}{2\pi} \int [\hat{V}_c(\Omega, \xi) f_c(\Omega, \tau) + \hat{V}_s(\Omega, \xi) f_s(\Omega, \tau)]$$
$$\times \mathrm{d}\Omega + \sum_{i=n,p,\tau,\theta} \hat{V}_i(\xi) f_i(\tau) \quad (4)$$

with operator coefficients, where the discrete eigenmodes f_n, f_p, f_τ, and f_θ represent perturbations of the soliton mean field due to changes in photon number, momentum, time and phase respectively; and f_c and f_s are the symmetric and anti-symmetric continuum eigenmodes that represent perturbation of the dispersive radiation in the fiber. Detailed analysis of these modes, along with their time-domain and frequency-domain forms, can be found in [8].

The ξ-dependent Hermitian operators $\hat{V}_i$, $i \in M \equiv \{c, s, n, p, \tau, \theta\}$, are obtained by projecting equation (4) onto the eigenmodes $\{\tilde{f}_i \,|\, i \in M\}$ of the equation adjoint to equation (3) which is obtained by reversing the sign of the $\Delta \hat{b}^\dagger$ term. The relevant orthogonality relations are $\langle f_i, \tilde{f}_j \rangle = \Delta_{ij}$, where the inner product is defined by $\langle f, \tilde{g} \rangle \equiv \mathrm{Re} \int f(\tau) \tilde{g}^*(\tau) \mathrm{d}\tau$; $\Delta_{ij} = \delta_{ij}$ in all cases except for $i = j \in \{c, s\}$, where $\Delta_{ij} = 2\pi\delta(\Omega - \Omega')$.

Defining the time-domain cosine quadrature operator $\Delta \hat{a}_c = (\Delta \hat{a} + \Delta \hat{a}^\dagger)/2$, we write the time-domain correlation function $G(\tau, \tau'; \xi) = 4\langle \Delta \hat{a}_c(\tau, \xi) \Delta \hat{a}_c(\tau', \xi) \rangle$ and the corresponding covariance function $C(\tau, \tau'; \xi) = G(\tau, \tau'; \xi) - 4\langle \Delta \hat{a}_c(\tau, \xi) \rangle \langle \Delta \hat{a}_c(\tau', \xi) \rangle$. Assuming that the filter $H(\omega)$ at the fiber exit is linear and imposing the realizability condition $0 \leqslant |H(\omega)| \leqslant 1$, we obtain the perturbation operator after the filter from the frequency-domain relation

$$\Delta \hat{a}_{\mathrm{out}}(\omega, \xi) = |H(\omega)| \Delta \hat{a}(\omega, \xi) + \sqrt{1 - |H(\omega)|^2} \hat{v}(\omega), \quad (5)$$

where $\hat{v}$ is a vacuum-state operator associated with the frequency-dependent loss due to the filter. The observed squeezing $S(\xi)$ is quantified by normalizing the output photon-number variance to the average output photon number:

$$S(\xi) = 1 + \frac{1}{4\pi^2 \langle \hat{N}_{\mathrm{out}} \rangle} \int \int \mathrm{d}\omega \mathrm{d}\omega' f_0(\omega) |H(\omega)|^2$$
$$\times C_N(\omega, \omega'; \xi) |H(\omega')|^2 f_0(\omega'), \quad (6)$$

where $C_N(\omega, \omega'; \xi) = C(\omega, \omega'; \xi) - 2\pi\delta(\omega - \omega')$ is the normally ordered part of the Fourier transform $C(\omega, \omega'; \xi)$ of the covariance function $C(\tau, \tau'; \xi)$, and $f_0(\omega) = \pi \mathrm{sech} \frac{\pi\omega}{2}$ is the Fourier transform of $f_0(\tau)$.

As stated by Levandovsky *et al* [7], in the case of the SSS the correlation function $G(\omega, \omega'; \xi)$ is equal to the covariance function $C(\omega, \omega'; \xi)$ because the perturbation at the fiber entrance ($\xi = 0$) is white coherent-state quantum noise. The observed squeezing in dB, given by $-10 \log S(\xi)$, is shown in figure 1 for the case of a band limited parabolic filter [9] $H(\omega) = 1 - \omega^2/\eta^2$, $|\omega| \leqslant \eta$, where the bandwidth η is adjusted to give a 10% loss. The analytical expression for the correlation function G in time domain, as well as the observed squeezing for other types of filters, are given in [7].

Now we turn to the analysis of the DSS. In order to render the problem tractable while retaining its essential physical features, we have made the following assumptions: (a) the filter
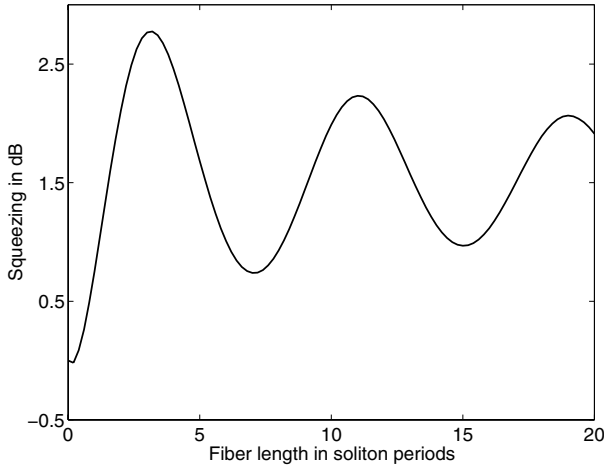
Figure 1. Observed squeezing (in dB) versus fiber length (in soliton periods) in the SSS. The filter has a band limited parabolic frequency response giving a 10% loss.

in the first stage is weak enough that the quantum-mechanical average of its output may be treated as the soliton mean field $a_0(\tau, \xi)$ plus a small perturbation $\Delta b_0(\tau, \xi) \equiv \Delta a_0(\tau)e^{i\xi/2}$; (b) all relevant frequency-domain quantities are narrowband[2], so that the number of photons at high frequencies that are cut off with a band limited filter is negligible, and the mean field can still be treated as a fundamental soliton; and (c) as the pulse propagates through the second stage, the quantum-mechanical average $\langle \Delta \hat{a}(\tau) \rangle$ remains independent of ξ and is equal to $\Delta a_0(\tau)$. This last assumption can be given precise mathematical meaning by ensuring that the maximum steady-state ($\tau \to \infty$) error which results from assuming that $\Delta a_0(\tau)e^{i\xi/2}$ is a solution of the linearized equation (3) is vanishingly small[3]. With these assumptions in place, the DSS becomes equivalent to the SSS, but with a squeezed-state input and an additional filter inserted between the soliton source and the entrance to the fiber.

Since the filter is assumed to be weak, we may write its transfer function in the form $H(\omega) = 1 + h(\omega)$, where $h(\omega)$ is small, so that $\Delta a_0(\omega) = h(\omega) f_0(\omega)$ is the frequency-domain form of the quantum-mechanical average $\langle \Delta \hat{a}(\tau) \rangle$ which, as we have assumed, is independent of ξ. Therefore, at the fiber entrance to the second stage, the soliton mean field is perturbed by an ideal squeezed state with the quantum-

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[2] The assumption that all the relevant frequency-domain quantities are narrowband (i.e. have bandwidths small compared to the baseband frequency) is equivalent to the time-domain slowly-varying envelope approximation. We invoke this assumption in order to make the numerics more efficient. A more thorough treatment would necessarily call for determining the exact form of the post-filter emergent soliton.

[3] The steady-state error is given by

$$\epsilon_{\text{steady}} = \lim_{\tau \to \infty} \sup_{\xi} \left| \frac{\partial f(\tau, \xi)}{\partial \xi} - \mathcal{L}[f(\tau, \xi)] \right|,$$

where $f(\tau, \xi)$ is the approximate solution of the c-number version of equation (3) and $\mathcal{L}[\cdot]$ is the linear operator acting on the (mean) perturbation on the right-hand side of equation (3). In our case, $f(\tau, \xi) = (1/\eta^2) \exp(i\xi/2)d^2\text{sech}\tau/d\tau^2$, and the error is given by

$$\epsilon_{\text{steady}} = \lim_{\tau \to 0} \frac{6 \, \text{sech}^3\tau \tanh^2 \tau}{\eta^2} = 0.$$

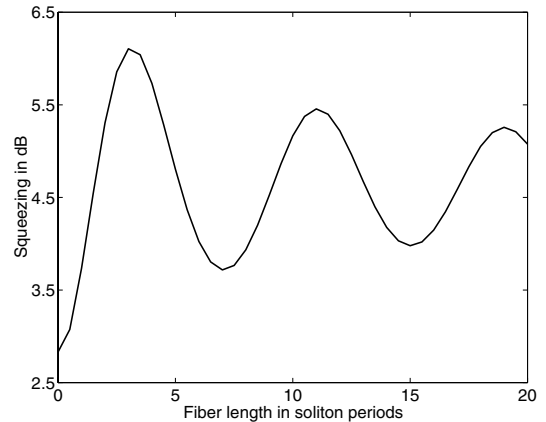In fact, the quantitiy under the limit sign rolls off very rapidly in τ.

Figure 2. Observed squeezing (in dB) versus second-stage fiber length (in soliton periods) in the DSS with the first stage generating maximally squeezed output (the first-stage fiber is three soliton periods long). The filter has a band limited parabolic frequency response giving a 10% loss.

mechanical average $\Delta a_0(\tau)$ and the squeezing parameter r determined by the fiber length and the filter transfer function of the first stage. It is then easy to see that the time-domain covariance function for the DSS is given by

$$C_{\text{DSS}}(\tau, \tau'; \xi) = e^{-2r}G_{\text{SSS}}(\tau, \tau'; \xi) - 4\Delta a_0(\tau)\Delta a_0(\tau'), \quad (7)$$

where $G_{\text{SSS}}(\tau, \tau'; \xi)$ is the correlation function for the SSS and $\xi = 0$ at the fiber entrance to the second stage. The observed squeezing $S_{\text{DSS}}(\xi)$ at the output of the DSS is then computed by substituting the normally ordered covariance function $C_{\text{DSS},N}(\omega, \omega'; \xi)$ into equation (6).

We have analysed numerically the following arrangement. If the fiber in the first stage is three soliton periods long, then with our particular filter the maximum squeezing is 2.8 dB, which corresponds to $r \simeq 0.32$. The initial perturbation $\Delta a_0(\omega)$ is given by $-\frac{\omega^2}{\eta^2} f_0(\omega)$ (for all frequencies $|\omega| \leqslant \eta$, a condition which is sufficient in accordance with our narrowband assumption). The maximally squeezed output of the SSS is then launched through the second stage. The observed squeezing at the output of the DSS with the first stage generating maximally squeezed output is shown in figure 2 as a function of the fiber length in the second stage, with maximum squeezing of 6.1 dB observed if the fiber in the second stage is three solition periods long.

The results of the preceding analysis suggest that by iterating the SSS it is possible to construct a wide variety of components for the generation and manipulation of squeezed states in optical networks for quantum communication and computation. As already mentioned, the SSS can be used to generate near-PNS states whose statistical parameters can be tuned by varying the pulse power and width. The main incentive to use the DSS for the generation of near-PNS states is the simplicity of implementation: in order to get more squeezing, we just add another stage to the SSS with another parabolic filter, thus avoiding the use of optimized filters whose shapes are complicated and difficult to implement [7]. In other words, compared to a SSS, one gets more squeezing out of a DSS while using less fiber and easily realizable filters.

As for the manipulation of squeezed states in such networks, the DSS, with more than a twofold increase

in squeezing (measured in decibels), renders an excellent illustration of how one can use fiber nonlinearity to enhance the noise statistics of light transmitted through the network. Such an enhancement would be needed, e.g. if the squeezing introduced previously was degraded by loss. To summarize, the capabilities offered by fiber nonlinearity need to be closely explored with such novel applications as quantum information processing in mind.

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